7 TIME & WORK

To start with, let's take an example. If 2 men take 10 days to build a wall then 1 man will take 20 days (not 5 days) to build the same wall.

Here in this example we can see that as the no. of men decreases, number of days increases. So, we can say that no. of men are inversely proportional to the no. of days taken to complete a certain work. i.e.

$M \propto 1/D$, where M & D are no. of men and no. of days respectively.

If we further break number of days to hours, then total hours = DH, where H are number of hours per day. Now, our formula becomes

$M \propto 1/(DH)$. ……………………..(1)

To elaborate it further, let's say M men take D days to build a room. Now if work is doubled (they have to build two rooms of same size) in same D no. of days, obviously they have to double their strength Or we can say that no. of men are directly proportional to the work to be finished. In mathematics, we can write it as

$M \propto W$, where W is the work to be finished……………..(2)

By combining (1) and (2), we get

$$M \; \alpha \; \frac{1}{DH} \times W$$

or $\frac{MDH}{W} = K$ Where K is constant of proportionality

$$\text{or } \frac{M_1D_1H_1}{W_1} = \frac{M_2D_2H_2}{W_2}$$

This is our general formula to solve time & work problems. It is also known as **Work Equivalence Method.** Majority of time & work questions can be divided into two types.

TIP

If a worker takes x days to finish a work (working alone) and second one takes y days to finish the same work (working alone), then together they will take $\frac{xy}{x+y}$ days to finish the same job.

Type I: Where **efficiency** of individual's is not mentioned (as in the above example): These are the cases where the man-days concept is utilized. Here the Rule of Fractions or the proportion (direct or indirect) concept can also be used.

Type II: Where the **efficiency** is not mentioned. Here the case of unit day's or one day's concept is utilized.

<u>Unit day concept</u>

If A does a work in x days, then work done in unit time = $\frac{1}{x}$.

Work done in unit time is known as efficiency of the worker or we can say, that if a worker takes less no. of days (than second worker) to finish a work, he will be more efficient.

Example

If 10 men can make 200 chairs working for 15 days at the rate of 8 hrs per day, how many chairs can 5 men make working for 10 days at the rate of 6 hrs a day?

Sol. 10 men working for 15 days at 8 hrs per day, put in 1200 man hours of work. They produce 200 chairs. If 5 men work for 10 days at 6 hrs per day, they put in 300 man-hours.
This is $(1/4)^{th}$ of the man hours put in by the first set of men.
Hence, the number of chairs that they can make = (1/4)x 200 = 50

<u>Alternative Method:</u> If the number of chairs produced by the second set of men is x.
By rule of fraction:

Required unit chair = x = $200 \times \frac{5}{10} \times \frac{10}{15} \times \frac{6}{8} = 50$.

Example

In a camp of 100 students, there is ration which lasts for 8 days. After the first 2 days, 50 more students join them. How long will the ration last now?

Sol. Suppose no more students had joined then the remaining ration would have lasted for 6 days. But since 50 more students are joining, it now lasts for fewer than 6 days. There are two ways of solving the problem :
<u>Approach 1:</u> Let each student consume *x* kgs per day. Hence if the ration lasts for 8 days when there are 100 students, the amount of ration left at the end of 2 days = (100) (6) (*x*) = 600 *x* kgs. This has to be now consumed by 150 students. Hence the number of days this would last = (600 *x*) / 150x = 4 days.

<u>Approach 2:</u> This is the method where we use proportions. If there are more people, then the ration should last for lesser time. The number of students now is (3/2) times the original number, hence the number of days it would last for would be (2/3) times the original number of days i.e. (2/3) (6) = 4 days.

One man works for10 hrs/day for 3 days to finish a work. A boy finishes the same in 5 days working 18 hrs/day. How many boys are required to do the job in the time taken by 1 man?

Sol. For the same job Man needs → $(10 \times 3) = 30$ hrs.

Boy needs → $(5 \times 18) = 90$ hrs.

Now it is evident that if the boys have to do the job in 30 hrs, then we will need $\frac{90}{30} = 3$ boys.

10 horses & 15 cows eat grass of 5 acres in a certain time. How many acres will feed 15 horses and 10 cows for the same time, supposing a horse eats as much as 2 cows.

Sol. 1 horse = 2 cows, 10 horses = 20 cows ⇒ 10 horses + 15 cows = 20 + 15 = 35 cows

15 horses + 10 cows = 40 cows. Now 35 cows eat 5 acres ⇒ 40 cows eat $5 \times \frac{40}{35} = 5\frac{5}{7}$ acres.

Tip: Here we have converted everything in terms of cows. You can even work in terms of horses also.

If 4 men, 5 women and 6 children do a work in 18 days, in how many days would 3 men, 6 women, and 8 children do it?

Sol. Identify the relationship (direct or indirect).

Man – Days is an Indirect relationship since more the men, lesser the no. of days required.

4 M, 5W, 6C → 18

3M, 6W, 8C→ ?

∴ *By rule of fraction:*

Number of days = $18 \times \frac{4}{3} \times \frac{5}{6} \times \frac{6}{8} = 15$ days.

Pipes and Cisterns

Pipes and Cisterns problems are almost the same as those of Time and Work problems. Thus, if a pipe fills a tank in 6 hrs, then the pipe fills $\frac{1}{6}$th of the tank in 1 hour. The only difference with Pipes and Cisterns problems is that there are outlets as well as inlets. Thus, there are agents (the outlets) which perform negative work too. The rest of the process is almost similar.

Inlet: A pipe connected with a tank (or a cistern or a reservoir) is called an **inlet**, if it fills the tank.

Outlet: A pipe connected with a tank is called an, **outlet**, if it empties the tank.

Toolkit

(i) If a pipe can fill a tank in x hours, then the part filled in 1 hour = $\frac{1}{x}$.

(ii) If a pipe can empty a tank in y hours, then the part of the full tank emptied in 1 hour = $\frac{1}{y}$.

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened = $\left(\frac{1}{x} - \frac{1}{y}\right)$.

(iv) If a pipe can fill a tank in x hrs and another can fill the same tank in y hrs, then the net part filled in 1 hr, when both the pipes are opened = $\left(\frac{1}{x} + \frac{1}{y}\right)$.

(v) If a pipe fills a tank in x hrs and another fills the same tank in y hrs, but a third one empties the full tank in z hrs, and all of them are opened together, the net part filled in 1 hr = $\left[\frac{1}{x} + \frac{1}{y} - \frac{1}{z}\right]$.

$\therefore$ Time taken to fill the tank = $\frac{xyz}{yz - xz - xy}$ hrs.

(vi) A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank = $\frac{xy}{y - x}$ hrs.

Example

Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Sol. Part filled by A alone in 1 hour = $\frac{1}{36}$

Part filled by B alone in 1 hour = $\frac{1}{45}$

$\therefore$ Part filled by (A + B) in 1 hour = $\left(\frac{1}{36} + \frac{1}{45}\right) = \frac{9}{180} = \frac{1}{20}$

Hence, both the pipes together will fill the tank in 20 hours.